

Optimal Design of a Reinforced Concrete Frame

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ABSTRACT: The optimal design of a reinforced concrete frame is dealt with by decomposition of the problem into a series of sub-frames with prismatic beams, columns and slabs. The optimization criterion chosen is the total cost of construction. An algorithm was developed to optimize the said problem and generate results for several different optimized sub-frames. The implementation of the algorithm in a computerized program is discussed in detail along with the parameters required to perform the necessary calculations. The optimization of a single sub-frame using the developed algorithm took under 3 seconds on a typical workstation. An analysis of the results produced shows that the columns and slabs can be optimized independently of the sub-frame. The optimal beam width is also found to be, generally, the minimum practical dimension possible while satisfying steel reinforcement spacing requirements. It is also shown that, for a fixed total span to be covered, a smaller number of spans is more economical. At current local market prices, using a lower concrete grade will generally result in a lower overall price and the optimization process can give savings of about 50%. A sensitivity analysis of the problem is also presented.

1 INTRODUCTION

New and different approaches to design have become possible through the increasing speed of computers and the mathematical tools of optimization theory. Processes which only had practical application in research, due to the computational time needed, are today possible for use in everyday design processes. A few years back, a great leap in the design process was possible since lengthy structural analysis and design could be carried out within minutes by having calculations automated instead of performing detailed hand calculations. This is not to say that hand methods were, or ever will be, discarded. Their use was simply changed, in certain instances, into a tool to get a rough feel for the numbers being dealt with. The computer then performed more detailed calculations. Today, however, the performance of computers is such, that calculations that used to take a matter of minutes, now take a matter of milliseconds. This therefore opens up the possibility of not only performing calculations for one particular problem, but extending the calculations to cover a range of solutions. One might argue that this has always been possible... it is just a matter of time.

This is very true; however time is probably the most important factor in the modern economics. Also, as

with all modern business, prices and technology are continually changing. Consequently, particular investigations regarding improvement of performance or efficiency may only be valid for a relatively short time frame.

Investigations are often needed for each particular scenario and availability of technology. Modern hardware, coupled with the right software, can cater for these needs within the required time frames. Such arguments hold for most design processes in the construction industry, not only to structural calculations.

Traditional design activity, even using computers, is based on postulating, appraising and modifying potential solutions in order to arrive at an acceptable form. Design by optimization employs numerical models of decision-making processes in order to generate direct prescriptive information on the nature of good solutions for the satisfaction of specified objectives. To some extent, it reverses the design process. Such procedures provide the potential for better design by encompassing a much wider range of possibilities, and offer the designer an opportunity to examine the implications of subjective decisions on the specified objectives.

Optimization theory coupled with cheap computational power provides the practical possibility to improve upon the design process without the need for impractical, more complex analysis

1.1 *Layout of the Dissertation*

This dissertation is broadly divided into two parts. The first part (sections 2 to 4) relates to the research leading to the proper definition of the problem. The second part (sections 5 to 6) relates to the work carried out to produce the required answers.

Section 2 gives a general background to optimization, algorithms and their application to engineering problems. Section 3 describes the current state of the art for optimal structural design, with particular attention given to research in optimal reinforced concrete design. Due notice is also paid to research which forms the basis of such applied optimal design. Section 4 describes the question of the dissertation concisely, showing the originality of the problem and utility of the answers.

Section 5 is a detailed description of how the problem was solved. In particular, it describes how a tool (in the form of a computer program) was developed to answer the question posed. This section also gives the results obtained and conclusions that may be drawn from them. Section 6 gives the main conclusions derived from this dissertation also giving an indication as to what further research could be conducted to follow up this work.

1.2 *General Objectives of the Dissertation*

The aim of this dissertation is to identify design relationships within a reinforced concrete structure's proportions and loading that lead to optimal, or near optimal design, given certain design constraints. So as to perform such a multi-dimensional analysis, an automated algorithm was developed to search for optimal solutions. From such data it is possible to test for the sensitivity of the optimal solution to a variation of the different variables. This optimization algorithm calls another algorithm developed to relate the structural design of reinforced concrete elements to their respective cost.

So as to achieve a successful and practical set of results the knowledge base for the program was derived from multiple disciplines:

Reinforced concrete structural design – to ensure a safe structure

Quantity surveying – so as to relate the forthcoming design to costs

Optimization – To develop efficient search algorithms.

2 PROBLEM STATEMENT

2.1 *Transposition of the real work problem*

The fundamental question in optimal building design is as follows:

What design will produce the building most suited to its purpose?

One might appreciate that the answer to such a question is extremely vast and a mathematically correct answer converges exponentially as new variables are introduced. This dissertation is concerned with a subset of such an answer. It is assumed that the architectural constraints are satisfied independently of the structural optimization process. Hence, cost of purely architectural elements affected by the structural optimization process are ignored. Realistic inclusion of such parameters would have limited the applicability of this exercise to a few particular cases, which is in contrast with the desired result. It should be noted, however, that the results of such an optimal process can also be used to guide the architectural design, by virtue of a sensitivity analysis of different scenarios.

Having set architectural constraints aside, the problem still remains considerable. One is left with an approximate geometrical description that may be satisfied in several ways. Two of these considerations are: the materials to be used and the way these materials are set into structural action. This work is only concerned with an in-situ reinforced concrete frame that is not required to resist lateral forces. It is hence assumed that shear walls, or effective bracing exists to provide lateral stability.

The optimization of a 3D reinforced concrete (RC) frame produces an effective, but particular solution to the problem. To overcome this problem, a 2D sub-frame will be optimized using a code design procedure, in attempt to give more widely applicable solutions. Such an exercise is very commonly used in practice, because the increase in efficiency provided by a fully 3D analysis is rarely justified by the increased computational effort. A 3D analysis does not provide the flexibility of a sub-frame analysis, which is particularly important as the building design progresses.

The problem may therefore be defined as follows:

The identification of patterns within the results of the optimization, by minimization of total direct cost, of a reinforced concrete continuous frame, through

the analysis of sub-frames consisting of a variable number of bays of prismatic rectangular beams with slabs and prismatic square columns.

The sub-frames are subject to different loading conditions, so as to avoid the production of data which is specific to one particular situation. The generation of various optimal solutions will enable the identification of patterns, in the combination of variables, that produce optimal, or near optimal designs.

2.2 *Limits to the scope of the problem*

The scope of the exercise is not to provide a fool proof design guidance, but to give an indication of the optimal proportions of the structure. Although the optimization process involves a detailed design routine, such an automated process may never replace the judicious design of an experienced structural engineer. This is particularly relevant to the design constraints which limit the applicability of the design results. Once the optimal proportions are obtained, it is assumed that the designer will use his/her expertise to interpret the information with respect to the particular exercise being undergone.

Practical optimization requires, not only good understanding of structural design, but also some appreciation of the mathematical optimization processes. The structural design involved must be accurate and careful, since situations may arise in optimal or near optimal structures, where the structure reaches several limit states simultaneously thereby increasing the risk of failure (Frangopol, D.M., & Moses, F., 1994).

For the purpose of this investigation, the assessment of the probability of failure of the entire structure as compared to the probability of failure of the particular elements is not tackled. This is because the analysis and design computations are not performed for the entire structure but for a sub-frame in accordance to BS8110 Part 1: 1997. The exercise therefore refers to a sub-set of elements from the entire building and the overall stability of the structure must be ensured apart from the results of the analysis performed here. A structural designer must therefore give due regard to the possibility of having alternative load paths among several other considerations.

The costs that will be considered in this exercise will be limited to construction costs, ignoring the effects of cost throughout the building life cycle such as maintenance and demolition costs.

3 VALIDITY OF THE PROBLEM

Although the optimization of a 2D sub-frame is essentially the scaling down of a 3D problem, it provides considerable advantages.

Probably the most important aspect of the sub-frame optimization lies within its flexibility. The exercise is just as flexible as the widely adopted sub-frame analysis enabling the designer to decompose the problem into smaller, more manageable parts. These can, in turn be distributed to different designers focusing on a smaller part of the entire project. Such an exercise could then be extended into a full decomposition method.

The simplification also eases the tackling of multiple load cases imposed on a structure, which is a major issue even in the traditional analysis of 3D structures. (Brohn, D., et al., 1995). General 3D solutions are too vast in terms of design variables to produce design charts or general guidelines, again hindering application in small projects. A 2D simplification overcomes this problem as well.

4 CONCLUSIONS

The problem stated was solved using an efficient computerized design algorithm. Patterns in the results of different optimal designs were found which can be used to decompose the problem further, giving the opportunity for increased processing speed.

The developed algorithm used to generate the required results can produce satisfactory results in very reasonable time frame. This also shows the versatility of the chosen problem, since such a tool can be used to produce optimal designs in real time. It is possible to incorporate such an algorithm into routine design applications, thereby opening the doors of optimal design for use by small practices.

Although the objective function for the optimization problem was cost, the results also show that the optimal values of certain variables are also those, which give the smallest volume of concrete. This particularly holds for the slab depth and the beam width. The minimum total volume of concrete does not, however, coincide with the minimum cost.

4.1 *Summary of findings*

The following is a listing of the main deductions that were made from the identification of patterns in the results generated:

- One may first optimize the columns of the sub-frame independently, then keep the column dimensions fixed while the remaining variables are optimized while maintaining very near optimal overall proportions.
- Assuming the desired design consists of slabs resting on beams (not a flat slab design), the optimal depth of the slab is dictated by the deflection constraint, over-ruling the sub-frame optimization. Therefore, the slabs can be optimally designed before, and independently of the sub-frame optimization, thereby further reducing the number of variables to be optimized.
- The optimal width of a beam is generally the minimum practical dimension it can have, regardless of span.
- It is more efficient in terms of cost, to keep the beam width to a minimum and increase the number of bar layers to accommodate the required reinforcement.
- For a fixed total span to be covered, a smaller number of spans is more economical.
- At current market prices, using a lower concrete grade will generally result in a lower overall price.
- The optimization process can give very considerable saving (of about 50%), even if the topology is fixed.
- The search algorithm may be refined for the column dimension, slab depth and beam depth; and slackened for the other variables so as to achieve greater accuracy in a shorter time.
- The total cost of the sub-frame is most sensitive to the column dimension, then to slab height and beam height.
- A tripling of the optimal percentage of redistribution from 10% to 30 % only resulted in a change of 2% in the total cost.

4.2 Future Research

It would be very interesting to speed up a sub-frame optimization algorithm by developing further the results of this dissertation. The algorithm could be used to produce the optimal design of a complete structure; the results of which could then be compared to a full 3D optimization routine. If the com-

promise in accuracy in favour of speed is acceptable, this could be used to direct further research and development of more accessible program suites for optimal design. It would also give a measurable justification for the sub-frame simplification.

It would also be worth comparing the results of the optimization of reinforced concrete with the optimization of different construction methods, such as pre-stressed concrete, post-tensioned concrete or steel frames. The range of efficient applicability of different methods of construction could then be derived.

5 REFERENCES

Brohn, D., Brown, D., Hernderson, R., & Rathbone, A. (1995) Modelling of Steel Structures for Computer Analysis. Berkshire: The Steel Construction Institute.

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